**Sample Paper 2 Model Solutions**

The solutions below are designed to give you an idea of how to approach the questions in the paper and, in some cases, a few alternative approaches. They are not an exhaustive list of possible methods and neither are they necessarily indicative of how much should be written to answer a question fully.

**__Question 1:__**

$$4 - (4 + 2x)$$
$$= 4 - 4 - 2x$$
$$= -2x$$

**__Question 2:__**

$$\frac{1}{3}(6 + 3x)$$
$$= \frac{6}{3} + \frac{3}{3}x$$
$$= 2 + x$$

**__Question 3:__**

$$5(p + qr)$$
$$= 5(4 + 3 \times -5)$$
$$= 5(4 - 15)$$
$$= 5 \times -11$$
$$= -55$$

**__Question 4:__**

$$(2x + 3)(x - 5)$$
$$= 2x^2 + 3x - 10x - 15$$
$$= 2x^2 - 7x - 15$$

**__Question 5:__**

$$3(x - 2) - 5(2 - x)$$
$$= 3x - 6 - (10 - 5x)$$
$$= 3x - 6 - 10 + 5x$$
$$= 8x - 16$$

**__Question 6:__**

$$x^2 - 5x - 6 = (x - 6)(x + 1)$$

**__Question 7:__**

$$(x - 2)^2 - (x + 2)(x - 2)$$

$$= x^2 - 2x - 2x + 4 - (x^2 - 2x + 2x - 4)$$
$$= x^2 - 4x + 4 - x^2 + 4$$
$$= -4x + 8$$

**Question 8:**

| | |
|---|---|
| $2x + 3y = 6$ | |
| $y = 4 - 3x$ | |
| $2x + 3(4 - 3x) = 6$ | substitute second equation into first |
| $2x + 12 - 9x = 6$ | |
| $-7x = -6$ | collect like terms and subtract 12 from both sides |
| $x = \frac{6}{7}$ | divide both sides by $-7$ |
| $y = 4 - 3 \times \frac{6}{7} = \frac{28}{7} - \frac{18}{7} = \frac{10}{7}$ | Substitute value of $x$ into second equation |
| $\frac{x}{y} = \frac{6}{7} \div \frac{10}{7} = \frac{6}{10} = \frac{3}{5}$ | |

**Question 9:**

$$\frac{3}{x} \div \frac{3}{x^2}$$
$$= \frac{3}{x} \times \frac{x^2}{3}$$
$$= \frac{3x^2}{3x}$$
$$= x$$

**Question 10:**

$$\frac{2x + 3}{x}$$
$$= \frac{2x}{x} + \frac{3}{x}$$
$$= 2 + \frac{3}{x}$$

**Question 11:**

$$\frac{x^2 + 4x + 4}{x^2 - 4}$$
$$= \frac{(x+2)(x+2)}{(x+2)(x-2)}$$
$$= \frac{x+2}{x-2}$$

**Question 12:**

$$\frac{3x+1}{6}-\frac{2x-3}{4}$$
$$=\frac{6x+2}{12}-\frac{6x-9}{12}$$
$$=\frac{6x+2-(6x-9)}{12}$$
$$=\frac{6x+2-6x+9}{12}$$
$$=\frac{11}{12}$$

**Question 13:**

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$
$$\frac{1}{f}=\frac{v}{uv}+\frac{u}{uv}$$
$$\frac{1}{f}=\frac{u+v}{uv}$$
$$f=\frac{uv}{u+v}$$

**Question 14:**

| | |
|---|---|
| $3 < -3x \leq 6$ | |
| $1 < -x \leq 2$ | divide both sides by 3 |
| $-1 > x \geq -2$ | multiply both sides by $-1$ (remember that the inequality signs reverse) |
| $-2 \leq x < -1$ | read right to left |

**Question 15:**

A: $RHS = 6a \times b = 6ab = LHS$ so this statement is true

B: $RHS = 6b \times a = 6ab = LHS$ so this statement is true

C: this statement is true

D: $RHS = 6a \times 6b = 6 \times a \times 6 \times b = 36ab \neq LHS$ so this statement is **false**

**Question 16:**

A: substitute in $(1, 1)$: $2(1) + 3(1) - 5 = 2 + 3 - 5 = 0$ so this statement is true

B: substitute in $\left(\frac{5}{2}, 0\right)$: $2\left(\frac{5}{2}\right) + 3(0) - 5 = 5 + 0 - 5 = 0$ so this statement is true

C: substitute in $\left(0, \frac{5}{3}\right)$: $2(0) + 3\left(\frac{5}{3}\right) - 5 = 0 + 5 - 5 = 0$ so this statement is true

D: this equation is equivalent to $3y = -2x + 5$

which is equivalent to $y = -\frac{2}{3}x + \frac{5}{3}$ so this statement is **false** (the gradient is $-\frac{2}{3}$ )

**Question 17:**

*Algebraic approach :*

Let $x$ be the cost of a bucket in pounds.

Let $y$ be the cost of a spade in pounds.

$$x + y = 1.5$$

$$x = 1 + y$$

So, by substituting the second equation into the first, we can say that $1 + y + y = 1.5$

$$2y = 0.5$$

$$y = 0.25$$

so the pencil costs $25$p.

*Alternative approach:*

Given that the bucket costs £1 more than the spade, the total cost would be $50$p if they both cost the same as the spade. Therefore the spade costs $25$p.

**Question 18:**

Let the width of the rectangle be $x$.

We can then deduce that $12 \times x = 48$ from which we deduce that $x = 4$.

So the perimeter of the rectangle is $2(4 + 12) = 32$ cm.

This means that the square has side length $\frac{32}{4} = 8$ cm.

Therefore, the square has area $8^2$ cm$^2$ $= 64$ cm$^2$.

**Question 19:**

$10$ km $= 1000000$ cm so the map scale is $1{:}\,1000000$.

Therefore the Linear Scale Factor is $10^6$ so the Area Scale Factor is $(10^6)^2 = 10^{12}$.

So the area on the map is $810 \div 10^{12} = 8.1 \times 10^{-10}$ km$^2$.

$1$ km $= 10^5$ cm so $1$ km$^2$ $= 10^{10}$ cm$^2$.

Therefore, the area on the map is $8.1 \times 10^{-10} \times 10^{10}$ cm$^2$ which is equal to $8.1$ cm$^2$.

*Alternative approach:*

$1$ cm on the map represents $10$ km in real life so an area of $1$ cm$^2$ on the map represents $100$ km$^2$ in real life. Therefore, the area on the map representing $810$ km$^2$ in real life is $\frac{810}{100} = 8.1$ cm$^2$.

**Question 20:**

Let the number of people originally on the bus be $x$.

If half of the people get off and then five get on, there will now be $\frac{1}{2}x + 5$ people on the bus.

If there are a third fewer than there were previously, there are now $\frac{2}{3}x$ people on the bus.

From this we can deduce that $\frac{1}{2}x + 5 = \frac{2}{3}x.$

$$3x + 30 = 4x$$

$$x = 30.$$

This means there must have been 30 people on the bus to start with.

*Check*: Half of 30 is 15, 15 plus 5 is 20. 20 is indeed two thirds of 30.

**Question 21:**

Let the spheres have radius $r$. Then the cylinder has radius $r$ and height $6r$.

So the volume of water only = volume of the cylinder – thrice the volume of a sphere

$$= \pi(r)^2(6r) - 3 \times \frac{4}{3}\pi r^3$$

$$= 6\pi r^3 - 4\pi r^3$$

$$= 2\pi r^3$$

so $2\pi r^3 = \pi r^2 h$ where $h$ is the height of the water only.

$$2\pi r^3 = \pi r^2 h$$

$$2r = h$$

$2r \div 6r = \frac{1}{3}$ so the water reaches $\frac{1}{3}$ of the way up.

*Alternative approach:*

The three spheres have total volume $4\pi r^3$ and the cylinder $6\pi r^3$.

So the spheres fill up $\frac{2}{3}$ of the volume and the water makes up $\frac{1}{3}$.

So the water alone would fill up $\frac{1}{3}$ of the height.

**Question 22:**

Let the amount of money shared be $8x$.

This means that, last year, the younger sister got $3x$ and the older sister $5x$.

This year, the younger sister needs to get $\frac{4}{3} \times 3x = 4x$ and so the older sister also gets $4x$.

Therefore, the ratio in which they share the money is $1:1$.

**Question 23:**

Jeremie crosses the finish line after 12 minutes and then after 24 and each subsequent multiple of 12.

So, in order to find the time that they cross simultaneously, we need the Lowest Common Multiple (LCM) of $12, 15$ and 18, for which we need the respective prime factorisations.

NB: listing multiples of each number is a possible, but very slow, way of doing this as well.

$$12 = 2^2 \times 3, 15 = 3 \times 5, 18 = 2 \times 3^2$$

so $\text{LCM}(12, 15, 18) = 2^2 \times 3^2 \times 5 = 2 \times 5 \times 2 \times 3^2 = 10 \times 18 = 180.$

In 180 minutes, they can each manage the following number of laps:

J: $180 \div 12 = 15$ K: $180 \div 18 = 10$ S: $180 \div 15 = 12$

So between them they run 37 laps, earning them $37 \times 20 = £740$.

**Question 24:**

Let the time in seconds for which they are cycling be $t$. Then Adrianna covers $13.5t$ metres and Jit covers $10.5t$ metres. They collectively cover $1000$ metres and so we can deduce this equation:

$$13.5t + 10.5t = 1000$$

$$24t = 1000$$

$$t = \frac{1000}{24} = \frac{125}{3} \text{ seconds}$$

Adrianna cycles $3m/s$ faster than Jit so covers an extra $\frac{125}{3} \times 3 = 125$ metres in that time.

*Alternative approaches:* Alternative approaches do exist but are effectively the same thing and so are not written out here. Two others you might use are:

- The speed at which they approach each other is $24m/s$ so you can work out how long they take.
- Adrianna and Jit cycle for the same length of time so they cover distance in the ratio $13.5: 10.5$ so you can divide $1000$ metres in that ratio and go from there.

**Question 25:**

We first establish that triangles $ABC$ and $XYC$ are similar:

| | |
|---|---|
| $< CXY = < CAB$ | corresponding angles are equal |
| $< CYX = < CBA$ | corresponding angles are equal |
| $< XCY = < ACB$ | same angle |

$|CX| = 6, |CA| = 9$ so the linear scale factor from triangle $ABC$ to $XYC$ is $\frac{6}{9} = \frac{2}{3}$.

Therefore the area scale factor from triangle $ABC$ to $XYC$ is $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

So triangle $XYC$ has area $27 \times \frac{4}{9} = 12$ cm$^2$ meaning the trapezium has area $15$ cm$^2$