WESTMINSTER SCHOOL
THE CHALLENGE 2023

MATHEMATICS II

Tuesday 25 April 2023

Time allowed: 1 hour 30 minutes

You will need a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

Write your answers in the answer booklet provided.

1 a i Multiply $\dfrac{3a}{7b^2}$ by $\dfrac{14b}{15a^2}$. Simplify your answer fully.

ii What would you subtract from $\dfrac{1}{2}q+1$ to make $\dfrac{1}{2}(q+1)$?

b Make T the subject of $\dfrac{4}{T}-\dfrac{3}{2Q}=\dfrac{1}{Q}$.

**c** Find the value of $\dfrac{x}{y}$, given that $\dfrac{x-2y}{3x+y}=\dfrac{4}{5}$.

2 Harry paid a total of £24.76 for thirteen bags of mini-eggs and six chocolate muffins. The chocolate muffins were sold for 79p each.

Harry later works out that he paid 7p for each individual mini-egg. How many mini-eggs were in each bag?

3 The rate of flow of the Amazon River is 219 000 cubic metres per second. The Earth's oceans have a surface area of 350 600 000 km^2 and an average depth of 3 900 m.

If the oceans were empty, and assuming that the Amazon's rate of flow remains constant, calculate the approximate number of years that it would take for the Amazon to refill the oceans.

4 The cylinder shown has height 14 cm and volume 625.77 cm^3.
Find the circumference of one of its circular ends.

14 cm
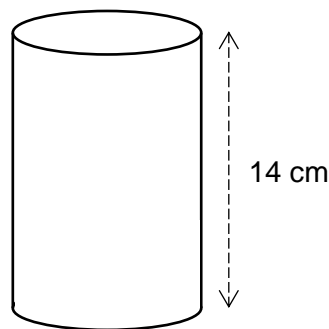
5 a In 2022, Aaron spent 42% of his pocket money on computer games. He spent £168.84 on computer games. How much pocket money did Aaron get in 2022?

b Bhavit's pocket money went up by 16% in each year from 2017 to 2022. In 2022 he received £352.86 in pocket money. How much did Bhavit receive in 2017?

c In 2015, Chin received £190.45 in pocket money. Between 2015 and 2022, Chin's pocket money increased by a fixed percentage P each year. In 2022, he received £315.96. What is the value of P?

d Davin got the same amount of pocket money as Chin in 2015. His pocket money went up by a fixed number of pounds and pence each year, until, in 2022, he once again received the same amount of pocket money as Chin. Which boy received more pocket money in 2020, and by how much?

6 A school expedition is rather badly organised. There are *n* children signed up for the expedition. One-fifth of them are left behind at school. Eight are mislaid at a service station on the way. Half the remainder are lost on the expedition. Five more are abandoned at a rest stop on the way back.

    a Find an expression in terms of *n* for the number of children who make it back to school.

In fact, only a third of the children who signed up for the expedition make it back to school.

    b Find the value of *n*.

7 How many gallons of a solution containing 18% acid must be mixed with six gallons of a solution containing 32% acid to make a solution containing 23% acid?

8 The diagram shows a trapezium which has been divided into two triangles.
The ratio   area *L* : area *R*   is equal to 11 : 15.
Find the perimeter of the trapezium.

area *R* cm^2

4.8 cm   7.3 cm

area *L* cm^2
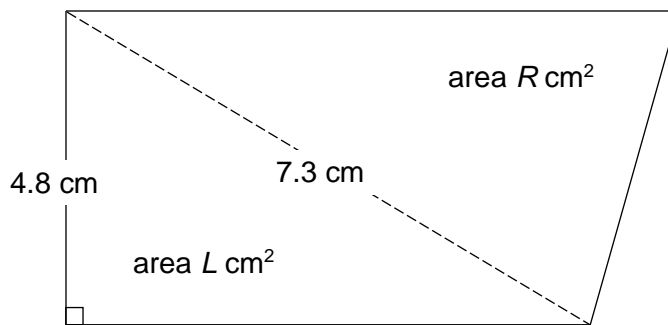
9 The diagram below shows a quadrilateral ABCD, in which AB = BC.
Some of the angles are given in terms of unknowns *x* and *y*.

A   B

$2x + y$ $4x + y$

$7x - 2y$
D

$4x - \dfrac{y}{2}$

50°

C
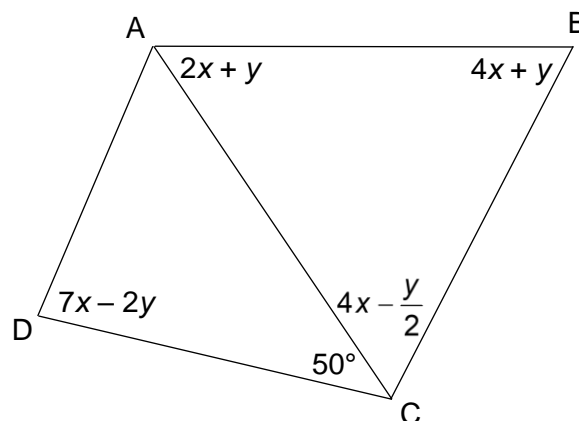
Justifying your answers carefully, prove that:

i AB is parallel to CD;
ii AC = CD.

QUESTIONS CONTINUE OVERLEAF

10 A car driver completes her journey from London to Cambridge via the motorway, taking 1 hour and 24 minutes. A motorcyclist takes the old road for the same journey: this is three miles shorter, but it takes 24 minutes longer because his average speed is 11 mph less than the car driver's. Find the length of each route.

11 An *Ideal Chocolate Orange* is a sphere of chocolate which can be split into 20 perfectly identical wedges, as shown in the diagram. Given that the total surface area of each wedge is 55 cm², find the volume of chocolate in a whole *Ideal Chocolate Orange*.

[A sphere of radius r cm has volume $\frac{4}{3}\pi r^3$ cm³ and surface area $4\pi r^2$ cm².]

sphere of chocolate…

… made up of 20 identical wedges
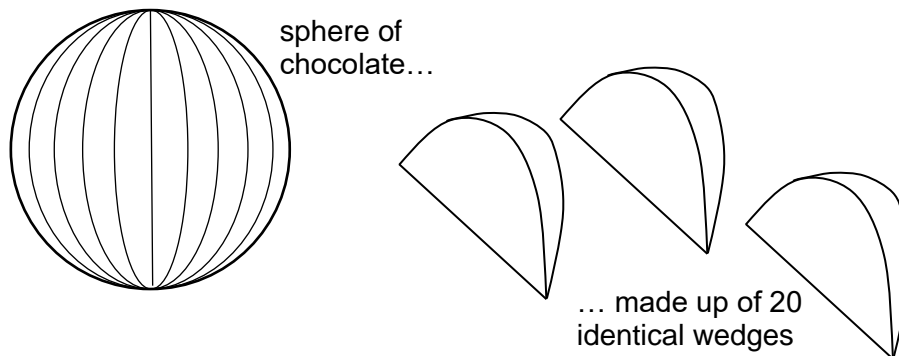
12 Solve the following simultaneous equations, giving the values of p and q correct to four significant figures. You may use your calculator to help, but you must show the algebraic steps of your work.

$$4.37p - 1.42q = 2.1$$

$$0.92p + 0.81q = 0.8$$

13 Kevin has a multi-coloured pen which can write in the seven colours of the rainbow: Red, Orange, Yellow, Green, Blue, Indigo, Violet.

He also has a circular piece of paper which has markings like a clock face, as shown in the diagram.

He makes a Red mark on the circumference at the position numbered "12". Then he moves his pen exactly 31 cm clockwise around the circumference and makes an Orange mark. Then he moves his pen another 31 cm clockwise around the circumference and makes a Yellow mark, and so on, with the colours continuing in the order given above until he has made one mark of each colour.


direction of pen movement
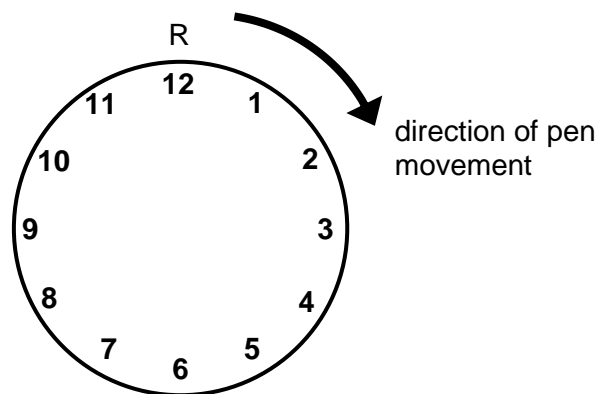
a If the radius of the paper is 8 cm, show that the Violet mark will be between the numbers "8" and "9".

b If the radius of the paper is 8 cm, in what sequence will the colours appear around the circumference of the paper?

c Find a different radius for the paper which would make the sequence of colours R-V-I-B-G-Y-O (working clockwise and starting from the position numbered "12").

QUESTIONS CONTINUE OVERLEAF

14 In this question, you might need to use the fact that, if p and q are positive numbers, then $\sqrt{\dfrac{p}{q}} = \dfrac{\sqrt{p}}{\sqrt{q}}$.

a The triangle shown has angles of 45°, 45° and 90°. The hypotenuse is x cm. Find an expression in terms of x for the lengths of the other two sides.
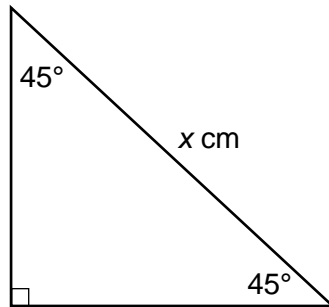
A kart racing circuit is laid out as shown in the diagram:
- A to B: a curve of radius 100 m, turning through an angle of 225°
- B to C: a straight section of length y m
- C to D: a curve of radius r m, turning through an angle of 135°
- D to A: a straight section of length y m
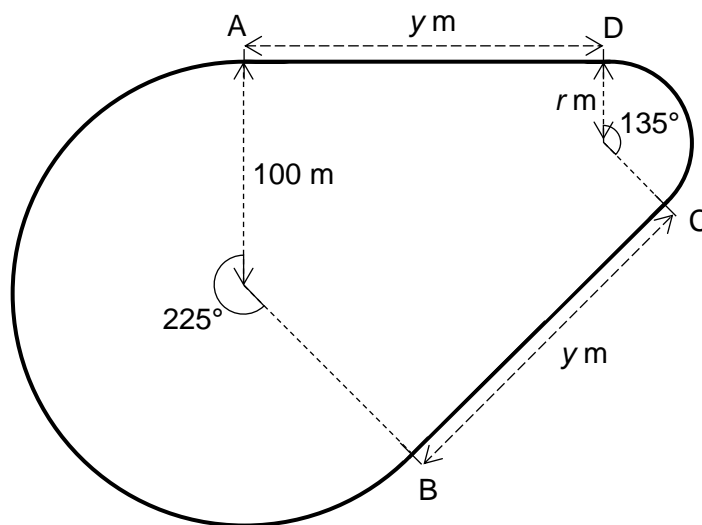
b If $y = 150$, find the value of r.

c If, instead, the total length of the circuit is 850 m, find the value of r.

END OF QUESTIONS

THIS PAGE HAS BEEN INTENTIONALLY LEFT BLANK

THIS PAGE HAS BEEN INTENTIONALLY LEFT BLANK